LP 11/6/12

Derivatives of Logarithmic and Exponential Functions

Find $\frac{dy}{dx}$.

1. $y = 4e^{1-2x}$

$4e^{1-2x}(-2) = \boxed{-8e^{1-2x}}$

2. $y = 2e^{\sqrt{x}} = 2e^{x^{1/2}}$

$2e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2} = \boxed{\frac{e^{\sqrt{x}}}{\sqrt{x}}}$

3. $y = (e^x + 3)^2$

$\boxed{y' = 2(e^x+3)e^x}$

$\boxed{y' = 2e^{2x} + 6e^x}$

4. $y = \ln(x + e^x)$

$y = \frac{1}{x+e^x}(1+e^x) = \boxed{\frac{1+e^x}{x+e^x}}$

5. $y = 2^{3x^2}$

$\boxed{y' = 2^{3x^2} \cdot \ln 2 \cdot 6x}$

6. $y = \frac{e^{4x}}{x}$

$\boxed{y' = \frac{x(4e^{4x}) - e^{4x}(1)}{x^2}}$ $\boxed{\frac{4xe^{4x} - e^{4x}}{x^2}}$

7. $y = \sqrt[3]{e^x} = (e^x)^{1/3} = e^{\frac{1}{3}x}$

$\frac{1}{3}(e^x)^{-2/3}e^x = \boxed{\frac{e^x}{3\sqrt[3]{e^x}^2}}$ $\boxed{\frac{e^{\frac{1}{3}x}}{3}}$

8. $y = \sin e^x$

$y' = \cos e^x \cdot e^x$

$= \boxed{e^x \cos e^x}$

9. $y = \log(\sqrt{x})$ $x^{1/2}$

$\frac{1}{\sqrt{x}\ln 10} \cdot \frac{1}{2}x^{-1/2} = \boxed{\frac{1}{2x\ln 10}}$

10. $y = 4^{2x}$

$y' = 4^{2x}\ln 4(2)$

$\boxed{y' = 2(4^{2x})\ln 4}$

11. $y = 3\log_4(2x)$

$3 \cdot \frac{2}{2x\ln 4} = \frac{6}{2x\ln 4} = \boxed{\frac{3}{x\ln 4}}$

12. $y = \log_2(x^2)$

$y' = \frac{1}{x^2\ln 2} \cdot 2x = \boxed{\frac{2}{x\ln 2}}$

$\ln y = 4x - \ln x$

$\frac{dy}{dx} = \left(4 - \frac{1}{x}\right)\left(e^{\frac{4x}{x}}\right)$

Derivatives of Logarithmic and Exponential Functions

Use logarithmic differentiation to find $\frac{dy}{dx}$.

13. $y = \sqrt{(x+5)(2x-1)}$

$\ln y = \frac{1}{2}(\ln(x+5) + \ln(2x-1))$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x+5} + \frac{2}{2x-1}\right)\sqrt{(x+5)(2x-1)}$

14. $y = \frac{x^2\sqrt[3]{x+2}}{6x+1}$

$\ln y = 2\ln x + \frac{1}{3}\ln(x+2) - \ln(6x+1)$

$\frac{dy}{dx} = \frac{2}{x} + \frac{1}{3}\cdot\frac{1}{x+2} - \frac{6}{6x+1}$

15. $y = \frac{(2x-1)^3}{\sqrt{x^2+x+1}}$

$\ln y = 3\ln(2x-1) - \frac{1}{2}\ln(x^2+x+1)$

$\frac{dy}{dx} = \left(\frac{6}{2x-1} - \frac{2x+1}{2(x^2+x+1)}\right)\frac{(2x-1)^3}{\sqrt{x^2+x+1}}$

$\frac{dy}{dx} = \left(\frac{2}{x} + \frac{1}{3(x+2)} - \frac{6}{6x+1}\right)\left(\frac{x^2\sqrt[3]{x+2}}{6x+1}\right)$

Find the equation of the tangent at the given point.

16. $y = -2x^2 + \ln x - 1$ at (1, -3)

$y' = -4x + \frac{1}{x}$

$y'(1) = -4+1 = -3$

$m = -3$

$y - -3 = -3(x-1)$

$y+3 = -3x+3$

$y = -3x$

17. $y = 6 - x^2 - \ln(2x+1)$ at (0, 6)

$y' = -2x - \frac{1}{2x+1}\cdot 2$

$y' = -2x - \frac{2}{2x+1}$

$y'(0) = -\frac{2}{1} = -2$

$y - 6 = -2(x)$

$y - 6 = -2x$

$y = -2x + 6$